CONCEPT OF GAMMA-RAY LASING ASSISTED BY NUCLEAR RECOIL EFFECT

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Abstract.

We present the general concept [1] of gamma-ray lasing which makes use of the kinematic shift and the splitting of the spectral lines of radiative gamma-transitions arising in an ensemble of free nuclei owing to recoil effect. These phenomena form a prerequisite for the formation of hidden population inversion with no excess of the number of excited emitters over the number of unexcited ones, for implementation of a broadband unperturbing x-ray pump in an unconventional 'two-level' scheme, for the manifestation of anisotropic unidirectional quantum amplification without any reflecting structures, etc. These attractive properties offer strong possibilities of staging an initial demonstration nuclear gamma-ray lasing experiment. This report, in a sence, is a continuation and development of the Final Technical Report prepared for European Office of Aerospace Research and Development in May 1997 (Project SPC-96-4033, Contract # F61708-96-W0179).

Contents

1. Introduction

«Decay of radium is a spontaneous event if the radium atom is an isolated system. However, this decay can be stimulated by the gamma-radiation field identical in frequency to the gamma-rays emitted in the decay of radium; the magnitude of this effect can be calculated by the Einstein formulas». This statement that stimulated emission of gamma-ray photons by excited nuclei is possible, made by A Eddington [2] long before the era of harnessing nuclear energy and the invention of optical lasers, reflects the complete awareness of the applicability of the laws of stimulated emission to the entire spectrum of electromagnetic waves and without exception to all types of quantum radiators.

Three decades later, C Townes and A Schawlow published the basic work [3] which gave impetus to the development of present-day quantum electronics of the optical range. They stated their confidence that lasing would never be extended past the boundary of the UV spectrum unless radically new approaches are introduced. This belief arose from the known fact of extremely fast rise of spontaneous emission rate with the photon frequency, which requires extremely high pump intensities to compensate for this rate in the formation of population inversion and to overcome the lasing threshold. Despite this discouraging prediction, the problem of stimulated gamma-radiation has attracted the attention of physicists for nearly 40 years. The reasons for this interest are obvious. The solution of this problem would allow a practical application of the fundamental regularities of stimulated emission of bosons, which are used to advantage in optical lasers, to a new class of quantum emitters - nuclei and even antiparticles. Next, it would open up a new energy range of coherent photons for contemporary science and technology - the kiloelectronvolt and, perhaps, even the megaelectronvolt range. Moreover, it put place in practice a new type of exothermal nuclear reaction: a chain reaction of stimulated radiative transitions of excited nuclei.

It is hoped that the advent of gamma-lasers will lend impetus to the emergence of *quantum nucleonics* [4] - new realm of physics, which is an extension of quantum electronics to the ultrashort-wavelength spectral domain around a new class of amplifying media.

It is clear that the authors of Ref. [3] intended the above negative prediction to apply to the electron transitions in atoms and ions. This prediction ignored the existence of metastable states of isomeric nuclei with an arbitrarily long spontaneous decay lifetime. For these neuclei the stated limitation ceases to be categorical. It may be that passing to metastable isomeric nuclei will form the basis for the sought-for radically new approach. However, time has softened to the pessimistic prediction as regards electron transitions, too: successful laser experiments have been reported in the X-ray range down to nanometer wavelengths (see, e.g. Ref. [5]).

Of course, the aforesaid by no means discredits the vital contribution of the authors of Ref. [3] to the foundation of quantum electronics and laser physics as realms of science and technology in their own right. Papers similar to Ref. [3] are of decisive importance even though they may not contain complete new, previously unknown results. Their significance is in coherent formulation of a self-consistent conceptual approach to a particular pressing problem, which opens the tangible way to its solution.

This report is an attempt to extend the ideas of quantum electronics beyond the UV boundary specified in Ref. [3] by presenting the general concept of a nuclear gamma-laser. The heart of the concept is a deliberate use of the recoil of free nuclei accompanying any event of absorption or emission of a gamma-ray photon.

As shown below, the use of nuclear recoil opens up, contrary to the standard scheme of optical lasers, entirely new possibilities inherent exclusively in radiative processes involving sufficiently hard photons:

* the possibility to establish a `hidden inversion' of state populations in an ensemble of nuclei, which appears without excess of the number of excited oscillators over the number of unexcited ones;
* the possibility to accomplish incoherent x-ray pumping of a nuclear ensemble in the «two-level» scheme, which is similar to the known three-level scheme of optical lasers but resorts to only two acting states of the laser transitions without resort to the third auxiliary level;
* the possibility of anisotropic quantum amplification of a unidirectional gamm-photon flux without any reflecting devices;
*the feasibility of internal modulation of photon losses accompanied by the emission of an intense gamma-ray pulse, in a sense similar to the giant pulse of an optical laser.

The above-listed and other remarkable features, which facilitate the progress towards the successful initial gamma-ray lasing experiment, reveal themselves only in a laser with free nuclei. That is why the concept under consideration differs fundamentally from the traditional Moessbauer approach in which nuclei are introduced into a solid matrix. But this circumstance simultaneously creates a need for a radical reduction in the unhomogeneous Doppler broadening of the emission line, which does not occur in the phonon-free solid version, to attain acceptable gain coefficients for the photon flux. This objective is to be accomplished by present-day techniques of deep cooling of atomic ensembles by the light pressure of optical lasers.

We shall therefore consider the concept of a gamma-ray lasing with a deeply cooled amplifying medium containing free nuclei. Its basic feature is the positive role of nuclear recoil in all radiative transitions involved in the quantum amplification of the gamma-photon flux.

In the following sections we shall point out the approaches to specific problems forming the integral problem. Also, we shall give examples of candidate isotopes and the general scheme of a conceivable experimental attempt to observe nuclear gamma-ray lasing. It is well to recall some commonly known facts to start with.

2. Amplification of the gamma-photon flux in stimulated transitions of excited nuclei

Some elementary estimates of the main factors controlling the amplification of a gamma-photon flux can be made by using a simple two-level model of a nuclear medium with resonance transitions with energy $E_0 = E_2 - E_1$, population densities n_2 and n_1 of the upper and lower levels (Fig. 1). In this model, the gain coefficient g assumes the form typical in quantum electronics:

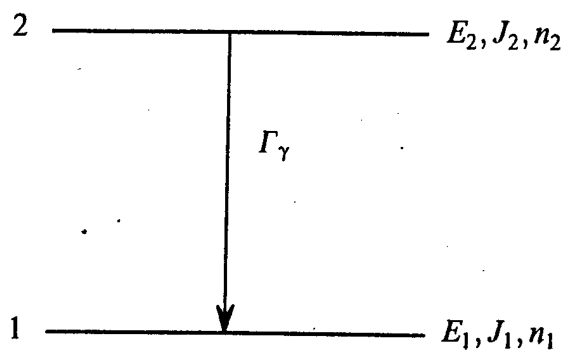

Figure 1. Two-level model of a nuclear amplifying medium (E_2 and E_1, J_2 and J_1, n_2 and n_1 denote the energies, the spins, and the population densities of states 2 and 1, respectively; Γ_γ is the intrinsic radiative transition width).

$$g = \sigma_0 \left(n_2 - n_1 \frac{2J_2 - 1}{2J_1 - 1} \right) \quad (1)$$

where

$$\sigma_0 = \frac{\lambda^2}{2\pi} \beta \quad (2)$$

is the resonance transition cross-section; J_2 and J_1 are the angular momenta of states 2 and 1; $\lambda = 2\pi c / \varpi$ is the transition wavelength; ϖ is the transition frequency; and c is the speed of light;

$$\beta = \frac{\Gamma_\gamma}{\hbar \Delta \varpi_{tot}} \quad (3)$$

is the ratio between the intrinsic radiative width Γ_γ of the $2 \rightarrow 1$ transition and the total emission linewidth $\hbar \Delta \varpi_{tot}$; allowing for all possible kinds of homogeneous and unhomogeneous broadening.

As is easy to see, to obtain a sufficiently high gain coefficient g care should be taken to increase two crucial factors - the linewidth ratio β, and the degree of population inversion or, in more exact terms, the positive difference Δn between the densities of the excited and unexcited nuclei:

$$\Delta n = n_2 - n_1 \frac{2J_2 - 1}{2J_1 - 1} \quad (4)$$

If it is assumed that the linewidth ratio $\beta = 1$ and the difference in nuclear densities corresponds to a rarefied gas density (2×10^{14} cm^{-3}), for photons with energy $\hbar\varpi = 10$ keV the gain coefficient becomes $g = 5 \times 10^{-3}$ cm^{-1} . In the case of exponential amplification during a single pass through an active medium of length $L = 500$ cm, we can expect a total gain $G = \exp gL = 12.2$.

It is quite clear that this overoptimistic estimate rests entirely on the assumption that $\beta = 1$. In reality, in the gamma-range and unless special precautions are taken the β ratio is many orders of magnitude lower than unity, which immediately reduces g to a completely unacceptable magnitude. In particular, in the case of free nuclei (gases, atomic or ion beams, etc.) the primary source of line broadening is the chaotic thermal motion of nucleus-bearing atoms, where $\hbar \Delta \varpi_{tot} \approx \hbar \Delta \varpi_D$ and the Doppler width of the emission line is

$$\hbar \Delta \varpi_D = 2\hbar\varpi \left[(2 \ln 2) \frac{kT}{Mc^2} \right]^{\frac{1}{2}} \approx 0.71 E_0 (T/A)^{\frac{1}{2}} \quad (5)$$

where M is the atomic mass; T is the absolute medium temperature; k is the Boltzmann constant; and A the number of nucleons in the nucleus. In Eqn (5) E_0 is expressed in terms of kiloelectronvolts, T is evaluated on the Kelvin scale, and $\hbar \Delta \varpi_D$ is expressed in terms of

millielectron volts. Hence it follows that, say, for E_0 = 10 keV, A = 100, $\Gamma_\gamma = 4\times10^{-9}$ eV (radiative lifetime $\tau_\gamma = 1\mu s$), and T = 80 K we obtain the ratio $\beta = 7\times10^{-7}$ and $\Delta\varpi_D / 2\pi = 1.5$ THz.

Thus in the solution of the problem of a nuclear gamma-ray lasing, the primary objective involves a radical increase in the linewidth ratio β, i.e. a reduction in the inhomogeneous broadening of the emission line.

3. Moessbauer version of a nuclear gamma-ray laser

Evidently, the Moessbauer effect, in which $\beta \to 1$, as a method for increasing β, came to the attention of the authors even of the very first Soviet and American proposals of a nuclear gamma-ray laser [6-12]. The possibility that $\beta \to 1$ was and still is in essence the only and a very important advantage of the Moessbauer version of the solution of the nuclear gamma-ray laser problem, in which nuclei are introduced into a cooled solid matrix with a high Debye temperature. Regrettably, this approach encounters serious difficulties and has not furnished positive results even at the conceptual level during the past 35 years, despite the vigorous activities of many researchers. Detailed references to their papers appear, for instance, in the monographic reviews [13 -20].

A comprehensive consideration of the internal contradictions of the Moessbauer approach to the nuclear gamma-ray laser problem is given in the recent exhaustive review by G C Baldwin and J C Solem entitled «Recoilless gamma-ray lasers» [20]. The title indicates that the authors deliberately restricted their study to precisely the Moessbauer version. In this review, these irreconcilable contradictions were termed the «graser dilemma». Briefly, the dilemma is as follows.

The conditions for the realisation of a Moessbauer transition can be maintained for nuclear states with not-too-narrow lines and with lifetimes of, say, $\tau_\gamma < 10\,\mu s$, for an intrinsic radiative width $\Gamma_\gamma > 4\times10^{-10}$ eV. The observation of the Moessbauer effect with lines of smaller width is hindered by various perturbing sources of broadening (inhomogeneity and defects in the crystal lattice, spatial nonuniformity of the temperature, random orientation of the nuclear magnetic momenta, etc., including the gravitational red shift).

It is significant that the density of excited nuclei should be relatively high because the spectral locations of emission and absorption lines coincide. Roughly speaking, to obtain $\Delta n > 0$ it is necessary that $n_2 > n_1$, i.e. that the degree of population inversion

$$\frac{\Delta n}{n} = \frac{\Delta n}{n_1 + n_2} > \frac{1}{2} \tag{6}$$

where n is the total nuclear density.

At the same time, the short lifetimes make it almost unfeasible beforehand to store excited nuclei in amounts sufficient to form above-threshold population inversion. Hence it follows that it is necessary to pump the nuclei already residing under the conditions of the Moessbauer

effect in a cooled solid matrix. Elementary estimates suggest that the required intensities of one or other types of pump (neutrons, X-ray photons, etc.) result in the immediate violation of the Moessbauer conditions owing to heating, impairment of the crystal structure, etc. Despite a wealth of diverse proposals [19, 20], including some quite sophisticated ones, today it is hardly possible to point out where and how this closed vicious circle can be broken. Research on the ways to implement the phononless Moessbauer version of a nuclear gamma-ray laser and the interesting proposals concerning the possibility to apply the principle of quantum amplification without inversion [21- 23] given in detail in Ref. [20] are beyond the scope of the present study. It is aimed at studying the alternative approach around free nuclei.

4. Nuclear recoil and hidden population inversion

A salient feature of the radiative processes in free nuclei not incorporated into a solid matrix is their recoil as a consequence of the laws of energy and momentum conservation in the emission or absorption of a gamma-photon with a relatively large momentum $\hbar\varpi/c$. For instance, to a free nuclei with $A = 100$ a photon with an energy of 10 keV is capable of imparting a tangible additional recoil velocity $c(\hbar\varpi/Mc^2) \approx$ 30 m/s. Thus, in any radiative transition a nucleus acquires the recoil kinetic energy

$$E_{rec} = \frac{(\hbar\varpi)^2}{2Mc^2} \approx \frac{E_0^{\ 2}}{2Mc^2} \approx 0.53\frac{E_0}{A} \tag{7}$$

(E_0 is expressed in kiloelectron volts, and E_{rec} in millielectronvolts). In emission this kinetic energy is drawn from the energy of a nuclear state

$$\hbar\varpi_e = E_0 - E_{rec} = E_0\left(1 - \frac{E_0}{2Mc^2}\right) \tag{8}$$

and in absorption from the photon energy

$$\hbar\varpi_a = E_0 + E_{rec} = E_0\left(1 + \frac{E_0}{2Mc^2}\right) \tag{9}$$

This signifies that the centres of the absorption and emission spectral lines are shifted by $\mp E_{rec}$ relative to the transition energy E_0 and by $2E_{rec}$ relative to each other. This phenomenon and its more complex forms considered below in sections 7 and 9 can conventionally be termed the kinematic shift and line splitting. Fig. 2 [24] depicts the shape of the spectral lines of an ensemble of free nuclei for $n_2 = n_1/2$ and $\hbar\Delta\varpi_D = 1.4E_{rec}$ [neglecting degeneracy; the dashed curves show the emission (positive ordinate axis) and absorption (negative ordinate axis) lines, the solid curve shows the summary spectrum].

An important conclusion is drawn from the kinematic shift of the emission and absorption lines: if the specified shift $2E_{rec}$ is comparable with the total linewidth $\hbar\Delta\varpi_{tot}$, there arise prerequisites for the formation of the so-called hidden inversion for the emitters belonging to a limited spectral interval within the total (unhomogeneous) width. In this case, the total inversion (4) is not required, i.e. the total number of excited nuclei need not be greater than the number of unexcited ones [25]. It is pertinent to note that quite efficient lasers with hidden

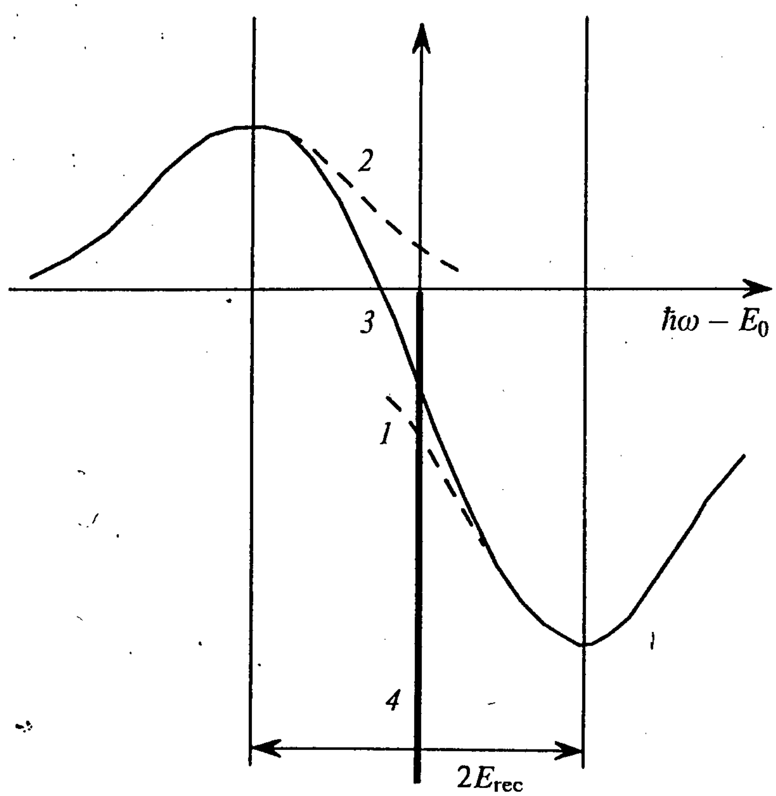

Figure 2. γ-Ray nuclear absorption (*1*) and emission (*2*) spectra (neglecting the degeneracy factor) for $n_2 = n_1/2, \hbar\Delta\omega_D = 1.4E_{rec}$, summary spectrum (*3*), and the Mössbauer absorption spectrum in the same case without total population inversion (*4*) (E_0 is the transition energy, E_{rec} is the nuclear recoil energy, and $\hbar\omega$ the photon energy) [24].

inversion are known to operate in the visible range. A case in point is injection semiconductor lasers in which a fairly narrow amplification spectral region and the region of strong absorption are spaced apart [24]. Another kind of hidden inversion was proposed in the method of forced Doppler modulation of emission and absorption nuclear lines [26].

Hidden inversion may be of interest in the solution of the gamma-ray laser problem to obviate the difficulties of the Moessbauer version [24, 27 - 30]. The quantitative criterion for the generation of hidden inversion in an ensemble of free nuclei with «temperature» T_h of the external translational degree of freedom, longitudinal with respect to the direction of the amplified photon flux, is the weakest of the inequalities [24]:

$$kT < E_{rec} \ln^{-1} \frac{n_1}{n_2}$$

$$\frac{n_2}{n_1} > \left[1 + \left(\frac{4E_{rec}}{\Gamma_\gamma}\right)^2\right]^{-1} \approx \left(\frac{\Gamma_\gamma}{4E_{rec}}\right)^2 \qquad (10)$$

By way of example, if $E_0 = 10$ keV, $\Gamma_\gamma = 1\mu eV$, and $A = 100$, then hidden inversion sets in even for a moderate cooling of nuclei: for $T_h < 8$ K if $n_1 = 2n_2$ and for $T_h = 0.5$ K if $n_1 = 10^7 n_2$.

The bold vertical line in Fig. 2 shows the absorption spectrum in the Moessbauer case for the similar relationship $n_1 = 2n_2$ when both the total and the hidden inversion are absent and the summary spectrum corresponds to pure absorption owing to coincidence of the spectral line positions.

Therefore, with a hidden population inversion the difference in Eqn (4) is merely equal to the density of excited nuclei,

$$\Delta n = n_2 \qquad (11)$$

and, accordingly, the gain coefficient [Eqn (1)] at the line centre

$$g = \sigma_0 \Delta n = \sigma_0 n_2 = \frac{\lambda^2}{2\pi} \beta n_2 \qquad (12)$$

depends neither on the density of unexcited nuclei n_1 nor on the relationship between the angular momenta J_1 and J_2. This important feature signifies that the condition $\Delta n > 0$ now does not require that approximate inequality (6) for the degree of inversion be necessarily fulfilled. The gain coefficient is positive ($g > 0$) for any finite n_2 and the ratio $\Delta n/n$ can be arbitrarily small if both weak inequalities (10) are fulfilled.

5. Criterion and threshold of amplification of a gamma-photon flux. The need for deep cooling

Relatively efficient reflectors in the gamma-ray range do not exist, and there is no reason to hope that the traditional laser scheme with a resonator feedback will be implemented in the

near future. Therefore, in what follows we will deal primarily with a travelling wave amplifier in the simplest version of a single-pass amplification of its inherent spontaneous radiation [31]. (Naturally, the results obtained below can be easily extended to the resonator version, if the mirrors become available.)

The intensity of spontaneous emission in a volume element of length dz and unit cross section in an extended channel of length L filled with excited nuclei with density n_2 is

$$dS_{sp} = \frac{n_2}{\tau_\gamma}\frac{\Delta\Omega}{4\pi}\frac{\Delta\varpi}{\Delta\varpi_{tot}}dz \tag{13}$$

where $\Delta\Omega$ and $\Delta\varpi$ are the solid angle and frequency interval covered by the preferred modes. As it propagates along the channel, the photon flux Φ' of spontaneous origin, which appeared in the volume element in the channel interval from z to $z+dz$, experiences amplification in the portion of the channel from z to $z = L$ in accordance with the equation

$$\frac{d\Phi'}{dx}dz = K(z)\Phi' dz \tag{14}$$

where

$$K(z) = g - \kappa n \tag{15}$$

κ is the total cross-section of photon losses due to the photoelectric effect, the Compton scattering by atomic electrons, etc. (see section 10); x=0 at point z and x=L-z at the exit of the channel. Integration of Eqn (14) from x = 0 to $x=L-z$ gives the output amplified photon flux $\Phi' dz$ that originates from the spontaneous photons emitted in the channel modes in the interval from z to $z + dz$. The total output flux Φ at $z = L$ results from the amplification of all spontaneous photons generated along the entire channel length L and is obtained by integrating $\Phi' dz$ over z from $z=0$ to $z=L$.

If all the quantities appearing in K in formula (15) are constant (K = const),

$$\Phi' = \frac{dS_{sp}}{dz}\exp[K(L-z)] \tag{16}$$

and

$$\Phi = \frac{\exp KL - 1}{KL}S_{sp} \tag{17}$$

where

$$S_{sp} = \frac{dS_{sp}}{dz}L \tag{18}$$

is the total number of spontaneous photons emerging into the preferred modes every second in a unit cross-section of the channel of length L. In the absence of the stimulated process these photons, which do not participate in quantum amplification but, on the contrary, are partly

absorbed in the channel characterised by $\kappa n > 0$, would represent at the output where $z=L$ a background spontaneous flux

$$\Phi_0 = \frac{1-\exp(-\kappa nL)}{\kappa nL} S_{sp} \tag{19}$$

which can be obtained from formula (17) by putting $g=0$.

That is why a trustworthy detection of quantum amplification of the gamma-photon flux as a result of stimulated nuclear transitions is possible only when Φ stands out considerably above the spontaneous background Φ_0:

$$G = \frac{\Phi}{\Phi_0} = \frac{\exp KL - 1}{1-\exp(K-g)L}\left(\frac{g}{K}-1\right) \tag{20}$$

which serves as the criterion for the presence of nuclear quantum amplification with threshold

$$G=1 \tag{21}$$

If $\kappa nL << 1$, then owing to the smallness of κ (see section 10),

$$G \approx \frac{\exp gL - 1}{gL} \tag{22}$$

For instance, $G= 3.2$ for $\lambda = 0.1$ nm, $\beta = 1$, $n_2 = 2.5 \times 10^{14}$ cm^{-3}, and $L = 5$ m.

Quantum efficiency can serve as another characteristic of stimulated emission. It is defined as the ratio of the output flux Φ [Eqn (17)] to the total intensity of spontaneous emission n_2L/τ_γ in the channel with a unit cross-section and length L:

$$Q = \frac{\Phi}{n_2L/\tau_\gamma} = \frac{\exp KL - 1}{KL}\frac{\Delta\Omega}{4\pi}\frac{\Delta\varpi}{\Delta\varpi_{tot}} \tag{23}$$

For isotropic (identical in all atomic translational degrees of freedom) cooling, the ratio $\Delta\varpi / \Delta\varpi_{tot}$ can be considered equal to parameter β [Eqn (3)] and the solid angle $\Delta\Omega/4\pi$ in an amplification channel of circular section with diameter D and length L (see section 6 below) can be estimated at $(D/4L)^2$. Then, in the case $\kappa nL << 1$, from formula (22) it follows that the quantum efficiency is

$$Q \approx G\left(\frac{D}{4L}\right)^2 \beta \tag{24}$$

More complex cases too are possible if $K \neq const$ along the length L, e.g. when excited nuclei come to the channel at $z= 0$ and gradually decay as they move lengthwise [31]. In this case the amplification criterion $G= 1$ and the concept of a threshold retain their significance.

From formulas (20) and (22) it follows that the linewidth ratio β continues to play a decisive role in the attainment of the above-threshold coefficient G under the conditions of hidden inversion. In particular, for gL <<1

$$\beta \approx 4\pi \frac{G-1}{\lambda^2 n_2 L} \tag{25}$$

A sufficiently high β can be attained only through deep «cooling» of the nuclear ensemble [24, 27] down to the longitudinal «temperature»

$$T_= = \frac{\Gamma_\gamma^2}{(16\ln 2)k\beta^2 E_{rec}} \approx 0.034 \frac{A}{\beta^2 E_0 \tau_\gamma^2} \tag{26}$$

where E_0 is expressed in terms of kiloelectronvolts, τ_γ in nanoseconds, and $T_=$ in millikelvins.

Several reference points can be marked off on the temperature axis. For instance, for E_0 = 10 keV and A = 100 the ratio $\beta = 0.5$ is attained for

$$\begin{aligned}
&\tau_\gamma = 3 \text{ ns} && \text{if } T_= = 10^{-4}\,\text{K} && \text{(DC)},\\
&\tau_\gamma = 10 \text{ ns} && \text{if } T_= = 10^{-5}\,\text{K} && \text{(SDC)},\\
&\tau_\gamma = 30 \text{ ns} && \text{if } T_= = 10^{-6}\,\text{K} && \text{(SRC)},\\
&\tau_\gamma = 1000 \text{ ns} && \text{if } T_= = 10^{-9}\,\text{K} && \text{(EC)}.
\end{aligned}$$

The formation of atomic ensembles in the specified temperature ranges is accessible for the modern practice of laser cooling of neutral atoms (see, e.g. review [32] and section 11). The above reference points (the abbreviations in parentheses denote different cooling techniques mentioned in section 11) signify that metastable states with nanosecond lifetimes too are noteworthy in deciding on a particular candidate nucleus.

6. Dimensions of the amplification channel. The number of excited nuclei

The shape of the amplification channel filled with a nuclear medium with a hidden inversion can be determined on the basis of the model of a single-pass amplifier adopted here and this channel represents an extended cylinder of length L and diameter D. An impression of the reasonable ratio between these dimensions can be gained from the following simple considerations [31]. Diffraction losses relating to a unit length of the channel are estimated as

$$\alpha_d \approx 2\frac{\lambda}{D^2}$$

It would be reasonable to impose the requirement that this quantity does not exceed appreciably the coefficient of unremovable losses in the channel material: $\alpha_d < \kappa n$. Hence it follows that the channel diameter is bounded below:

$$D > \left(2\lambda/\kappa n\right)^{\frac{1}{2}} \tag{27}$$

In the gamma-ray range this diameter proves to be extremely small, e.g. D > 0:015 cm for $\lambda =$ 0:124 nm and $\kappa n = 10^{-6}$ cm^{-1}.

In circumstances where the resonator mode is undeveloped (in the absence of mirrors) there arises a constraint that the photon beam does not diverge beyond the exit channel aperture on passing through distance L:

$$L \le \frac{D^2}{\lambda} \tag{28}$$

which is not a prohibitively rigorous constraint (in the above example L<200 m).

Another limitation on the channel length arises from the fact that a cooled nuclear ensemble is in most cases a supersaturated atomic vapour with a tendency to condense. The characteristic condensation time can be estimated by the atomic mean free time

$$\Delta t_c \approx \left[\sqrt{2} n \sigma_c v_T\right]^{-1} \tag{29}$$

where σ_c is the collision cross-section, and v_T is the thermal velocity of the cooled atoms. Let the cooled atoms arrive at the channel input with an average longitudinal translational velocity $v_0 > v_T$. To avoid condensation during their transportation along the channel, its length should satisfy the inequality

$$L < v_0 \Delta t_c = \frac{1}{n\sigma_c}\left(\frac{T_0}{2T_=}\right)^{\frac{1}{2}} \tag{30}$$

where T_0 is the temperature corresponding to velocity v_0. For instance, for $\sigma_c = 10^{16}$ cm^2, n=10^{14} cm^{-3}, and $T_0/T_= = 100$, the channel length L should not exceed 700 cm.

The minimum channel volume $V = \pi D^2 L/4$ is small even for appreciable lengths L (e.g. $V = 2\times10^{-2}$ cm^3 for L= 100 cm and D= 0.015 cm). Of course, the channel volume and diameter may be increased beyond the specified minimum values, if need be, to improve the output laser parameters. An important point is that the total number of excited nuclei $N = n_2 V$ in the channel of the minimum volume too may be moderately large even when their density is relatively high (referring to the above example, $N = 2\times10^{10}$ for $n_2 = 10^{12}$ cm^{-3}).

Therefore, in the model adopted the amplifying medium is an extended filamentary channel with relatively small diameter and volume. This shape is a prerequisite for staging primary experiments with a fairly small number of excited nuclei.

7. Pumping. 'Two-level' scheme

In the past decades a wealth of more or less sophisticated ways of preparing an ensemble of excited nuclei (i.e. ways of pumping) was proposed [20]. The most promising of these are eventually based on the two elementary processes - resonance absorption of a hard photon, and neutron radiative capture. The latter process could be rather efficient for some of the few nuclei with an anomalously large capture cross-section [20]. However, its use involves the difficult to achieve dissipation of the excess nuclear energy and momentum. Moreover, the availability of the appropriate neutron sources is dubious.

«Optical» pump, i.e. the excitation of nuclei in the absorption of hard photons (e.g. from an incoherent x-ray source) achieved by the conventional three-level scheme (Fig. 3a), which is used extensively in lasers of the optical range, is not without its drawbacks [20]. Suffice it to say that the intricacy of choosing the nuclei with the transition structure satisfying the requirements of a three-level scheme.

In addition to all these considerations, in attempting to operate any pump scheme we should remember that the pump will act on a previously prepared «cooled» nuclear ensemble. This occurs because the lifetime of the laser transition, which lies in the nanosecond range, is short (see section 5) and virtually rules out the reverse sequence of the operations of «cooling» and pumping. Therefore, apart from other features, the pump should be unperturbing, i.e. it should not break the monokinetic character of the «cooled» nuclear ensemble. In this connection, of some interest are the possibilities offered by the nuclear recoil considered above [33]. It turns out [34] that recoil in the photon absorption by a nucleus relieves the «optical» pump scheme of having to include the third (auxiliary) level, which is inevitably present in the pump scheme of optical lasers.

The kinematic shift discussed above (see section 4) of the absorption and emission lines arising from nuclear recoil transforms, in a sense, the two-level system into some analogue of a three-level one, since we are led to take into account the transfer of the energy and momentum to nuclear translational degrees of freedom. This opens up the possibility of pumping by relatively broadband radiation without the participation of a real auxiliary third level. In this scheme (Fig. 3b) the pump process involves absorption of photons with the energy shifted up [Eqn (9)] in going from level 1 to level 2. If the pump radiation band with frequency ϖ_p is limited below, so that $\hbar\varpi_p > \hbar\varpi_e$, excited nuclei can be stored in the same level 2 to the point of formation of true inversion, since the energy of emitted photons [Eqn (8)] is shifted down relative to the energy E_0 and their emission cannot be stimulated by the pump radiation.

Thus, level 2 splits kinematically into two levels and simultaneously plays the roles of the third and the second levels of the traditional scheme. As will be shown below, the formation of hidden inversion does not require that the pump band be bounded below. Attention is drawn to the important advantage of the «two-level» scheme over the conventional three-level scheme as regards energy efficiency ($\hbar\varpi_e/\hbar\varpi_p \approx 1$). This ratio is distinctly smaller than unity in the three-level scheme.

In the simplest version [34] of the «two-level» pumping, a beam of unexcited nuclei propagates with average velocity v_0 together with a collinear pump photon beam along the positive z-axis. Assume that the nuclear beam was subjected previously to monokinetisation and the pump is unperturbing (see below). Then we may assume that the low dispersion of the nuclear beam allows us to represent its velocity distribution, which arises due to the absorption

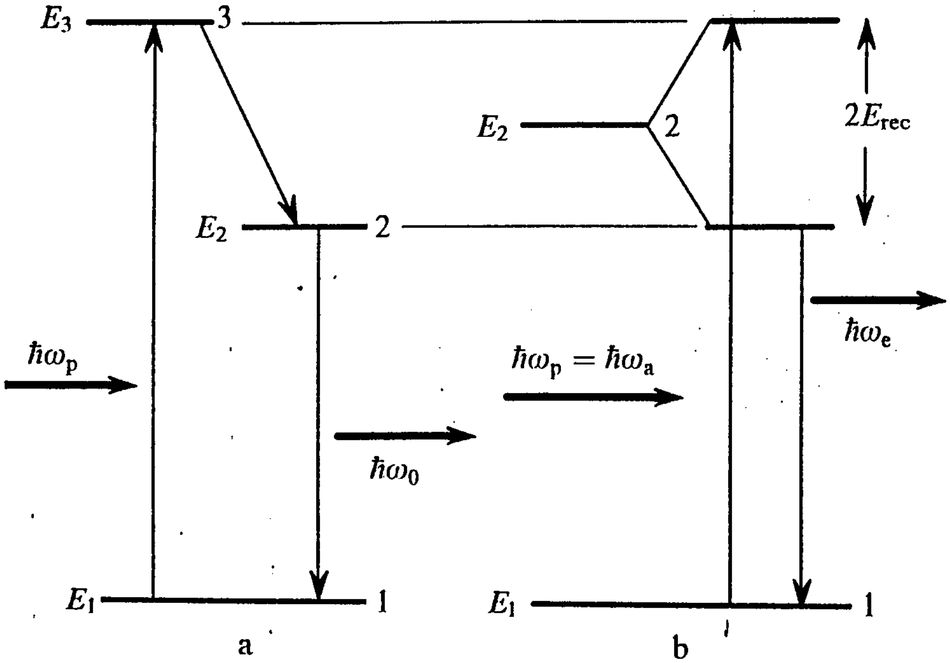

Figure 3. Conventional three-level scheme of optical pump (a) and 'two-level' scheme of optical pump in the γ-ray range as a result of the kinematic splitting of absorption and emission lines (b) (E_{rec} is the nuclear recoil energy, $\hbar\omega_p = \hbar\omega_a$ is the pump photon energy, and $\hbar\omega_e = \hbar\omega_0$ is the laser photon energy).

and stimulated emission of photons, as a set of discrete lines. In the comoving coordinate system, the initial distribution comprises a single zero-velocity line $\Delta v_0 = 0$. In every radiative transition involving absorption or emission of a photon with energy $\hbar\varpi$, which is collinear with the nuclear beam, a nucleus undergoes a positive or a negative velocity increment

$$|\Delta v| = \frac{\hbar\varpi}{Mc} \approx 2c\frac{E_{rec}}{E_0} \tag{31}$$

where approximate equality corresponds to neglect of the photon energy shifts [expressions (8) and (9)] relative to the transition energy E_0, which give a correction in the next order of smallness. Hence, when considering the kinematic shift of any transition, allowance should be made not only for the photon energy shifts by the recoil energy E_{rec}, but for the Doppler correction

$$E_0\frac{\Delta v}{c} \approx 2E_{rec} \tag{32}$$

as well, corresponding to the past radiative history of the nucleus.

The starting point for the transition cascade is the absorption of a pump photon with the energy

$$\hbar\varpi_p = \hbar\varpi_{a0} = E_0 + E_{rec} \tag{33}$$

by a nucleus belonging to the $\Delta v_0 = 0$ velocity group denoted by index $r = 0$. Consequently, there appears a new group $r = 1$ of excited nuclei with $\Delta v_1 = \Delta v_0 + \Delta v = \Delta v$. The nuclei of this group can emit photons, including those in the $\pm$ z-directions with the energy

$$\hbar\varpi_{e,\pm1} = \left(E_0 - E_{rec}\right)\left(1 \pm \frac{\Delta v}{c}\right) \approx E_0\left[1 + \frac{E_{rec}}{E_0}(\pm 2 - 1)\right] \tag{34}$$

Naturally, the photons emitted in the + z-direction are capable of stimulating the emission of similar photons by the excited nuclei of the $r = 1$ group. Moreover, according to formulas (32) and (34), these photons have the same energy $\hbar\varpi_{e,+1}$ as the pump photons [Eqn (33)] and, consequently, join their flow, are at resonance with the unexcited nuclei of the $r = 0$ group, and are capable of exciting these nuclei. After emitting a photon in the + z-direction, the nucleus acquires additional velocity $-\Delta v$ and returns to the initial $r = 0$ group with the velocity $\Delta v_1 - \Delta v = \Delta v_0 = 0$. In effect, the events listed above are no different (with the exception of the intermediate recoil effects resulting eventually in the zero effect) from the usual interaction of a two-level atom with resonance radiation which merely brings about some increase in the excited state density with the tendency for its saturation.

The course of events is different when nuclei of the $r = 1$ group emit photons in the - z-direction, i.e. opposite to the direction of the pump photons. The Doppler-shifted energy of these photons [Eqn (34)] is

$$\hbar\varpi_{e,-1} \approx E_0 - 3E_{rec} \tag{35}$$

Their emission results in the emergence of a new group of unexcited $r = 2$ nuclei with the velocity

$$\Delta v_2 = \Delta v_1 + \Delta v = 2\Delta v \tag{36}$$

The photon energy $\hbar\varpi_{e,-1}$ [Eqn (34)] differs from the energy of resonance absorption by the unexcited nuclei of the $r = 0$ group by

$$\hbar\varpi_{e,-1} - \hbar\varpi_{a0} = -4E_{rec} \tag{37}$$

This indicates the origin of the hidden population inversion between the nuclei of the $r = 1$ and $r = 0$ groups in a monokinetic beam, irrespective of their total density ratio. Conversely, the nuclei of the $r = 2$ group are capable of absorbing photons with the energy $\hbar\varpi_{e,-1}$, because

$$\hbar\varpi_{a,-2} \approx E_0 - 3E_{rec} = \hbar\varpi_{e,-1} \tag{38}$$

and consequently no hidden inversion exists between the $r = 1$ and $r = 2$ nuclear groups.

In the usual case of a broadband pump, when its upper frequency limit is sufficiently far away, the nuclei of the $r = 2$ group in their turn are subject to excitation by photons with the energy

$$\hbar\varpi_{a2} \approx E_0 - 5E_{rec} \tag{39}$$

and form the next group $r = 3$ of excited nuclei with the velocity

$$\Delta v_3 = \Delta v_2 + \Delta v = 3\Delta v \tag{40}$$

A continuation of this sequence of the events of absorption and emission of photons with wave vectors aligned with the $\pm$ z-axes gives rise to the discrete velocity distribution

$$\Delta v_r = |r|\Delta v \quad r = 0, \pm 1, \pm 2 \ldots \tag{41}$$

and the discrete line spectrum of kinematically shifted absorption resonances

$$\hbar\varpi_{ar} = E_0 + (2r+1)E_{rec} \quad r = 0, \pm 2, \pm 4 \ldots \tag{42}$$

and emission resonances

$$\hbar\varpi_{er} = E_0 + (2r-1)E_{rec} \quad r = \pm 1, \pm 3 \ldots \tag{43}$$

Here r = 0, 2, 4, . . . and r = 1, 3, 5, . . . are, respectively, the group numbers of unexcited and excited nuclei and also the numbers of absorption and emission lines, the signs «$\pm$» corresponding to the wave vectors of the photons propagating in the $\pm$ z-directions. The transition cascade considered above is illustrated in Fig. 4.

From formulas (42) and (43) it follows that, whatever the bounds of the pump band, hidden inversion arises between any nuclear velocity groups, with the exception of pairs for which

$$\hbar\varpi_{er_e} = \hbar\varpi_{ar_a} \quad i.e. \quad r_e - r_a = 1 \tag{44}$$

For these selected pairs of nuclear groups and for the photon beam in the -z-direction, excited nuclei belong to velocity groups with numbers $|r|$ that are smaller in modulus than the unexcited ones, i.e. to the earlier stages of the transition cascade. Since the group populations should evidently have the tendency to decrease with increasing $|r|$, this opens up the possibility for the formation of true inversion for the photons propagating in the -z-direction.

As noted earlier, a monokinetic («cooled») beam of free nuclei with a minimised relative inhomogeneous linewidth $(\beta \to 1)$ of the radiative transition is subject to an unperturbing pump. This signifies that the spectral perturbations $\delta\varpi$ introduced into the previously prepared «cooled» beam due to the radiative transition cascade during «two-level» pumping should be smaller than the initial inhomogeneous width:

$$\hbar\delta\varpi << \frac{\Gamma_\gamma}{\beta} \tag{45}$$

The possible sources of such perturbations are as follows. The absorption or the emission of photons with energies in the range of the linewidth Γ_γ/β gives rise to scatter $\delta(\Delta v_1)$ in the nuclear velocity increment and to the additional Doppler broadening

$$\delta\varpi_1 \approx \frac{\varpi\Gamma_\gamma}{\beta Mc^2} \tag{46}$$

of the nuclear resonance corresponding to this scatter. Condition (45) is satisfied by the self-evident strong inequality $\hbar\varpi << Mc^2$. It should be remembered that even with a broadband pump the energy of the absorbed photon lies within the linewidth Γ_γ/β owing to the resonance nature of absorption.

The divergence of the absorbed photon beam measured by plane angle $\Delta\phi$ reflects on the scatter of the longitudinal components of the photon momenta, which also give rise to additional Doppler broadening of the nuclear resonance by the quantity

$$\delta\varpi_2 = 2\varpi\frac{\hbar\varpi}{Mc^2}\left(1-\cos\frac{\Delta\phi}{2}\right) \approx \varpi\frac{\hbar\varpi}{Mc^2}\left(\frac{\Delta\phi}{2}\right)^2 \tag{47}$$

and condition (45) is fulfilled if

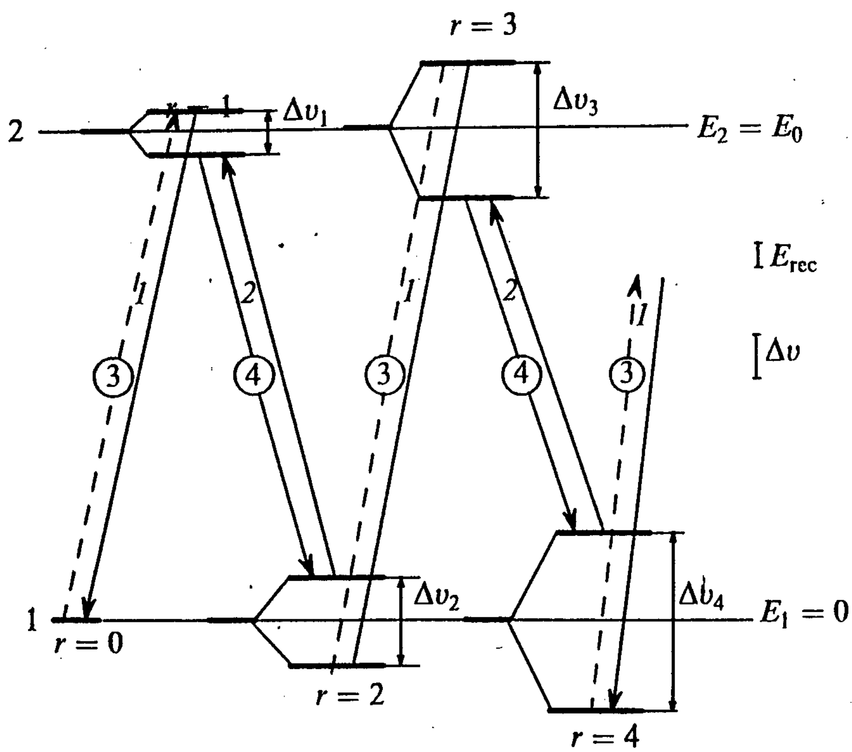

Figure 4. Transition cascade and velocity distribution of nuclei for a collinear pumping in the 'two-level' scheme (transitions with absorption and emission of photons in the $+z$-direction (*1*) and $-z$-direction (*2*), transitions with absorption of pump photons (*3*), stimulated transitions (*4*), nuclear velocity r groups). Shown at the right of the Figure are the scales of kinematic splitting E_{rec} and velocity deviation Δv of every group.

$$\left(\Delta\phi\right)^2 << 4\frac{Mc^2}{\hbar\varpi}\frac{\Gamma_\gamma}{\beta\hbar\varpi} \tag{48}$$

For instance, $\beta(\Delta\phi)^2 << 0.01$ sr for $E_0 = 10$ keV, $\tau_\gamma = 1$ ns, and $A = 100$.

The same divergence determines the scatter of lateral components of the photon momenta and is responsible for the second-order Doppler broadening

$$\delta\varpi_3 = \frac{\varpi}{2}\left(\frac{\hbar\varpi}{Mc^2}\right)^2 \sin\frac{\Delta\phi}{2} \approx \frac{\varpi}{2}\left(\frac{\hbar\varpi}{Mc^2}\right)^2\left(\frac{\Delta\phi}{2}\right)^2 \tag{49}$$

In this case inequality (45) is satisfied even more easily than in the previous case:

$$\left(\Delta\phi\right)^2 << 8\left(\frac{Mc^2}{\hbar\varpi}\right)^2\frac{\Gamma_\gamma}{\beta\hbar\varpi} \tag{50}$$

There are more rigorous limitations on the divergence of a pump photon beam when it is incident normally on the nuclear beam (i.e. if the collinearity is abandoned completely). In this case, the first-order Doppler broadening

$$\delta\varpi_4 = 2\varpi\frac{\hbar\varpi}{Mc^2}\sin\frac{\Delta\phi}{2} \approx \varpi\frac{\hbar\varpi}{Mc^2}\Delta\phi \tag{51}$$

requires that the inequality

$$\Delta\phi << \frac{Mc^2}{\hbar\varpi}\frac{\Gamma_\gamma}{\beta\hbar\varpi} \tag{52}$$

be fulfilled to satisfy inequality (45). Therefore, fulfilment of inequalities (48), (50), and (52) makes the pump unperturbing.

Overwhelmingly large perturbations to the velocities of «cooled» nuclei are introduced by electron detachment in the ionisation of nucleus-bearing atoms under intense pump irradiation. This process in effect expels the resulting ions from the operating nuclear beam, from which they can be removed by applying a lateral electric field (see section 10). The relative nuclear losses are estimated from the ionisation-to-nuclear level excitation cross-section ratio and may amount to a few percent.

It should be emphasised that the resonance nature of the pump - nuclei interaction in combination with the absence of any additional absorption centres other than the active nuclei eliminates the problem of overheating of the active medium, which is one of the central problems in the conception of solid-state gamma-ray lasers.

8. Density of excited nuclei in the «two-level» pump scheme

In the evaluation of attainable densities of excited nuclei [34] the cascade of nuclear radiative transitions can be terminated. This is so because, in particular, the transition to the next r group of unexcited nuclei requires a highly intense emission of photons of the previous spectral line $\hbar\varpi_{e,-(r-1)}$, i.e. in effect a developed stimulated process possible only when the amplification threshold is sufficiently exceeded. For the moderate realisable amount of excited nuclei, the threshold can possibly be exceeded only for the first line with hidden inversion and the photon energy $\hbar\varpi_{e,-1}$. Hence, in what follows it would suffice to retain out of the entire succession only the radiative transitions in the nuclei belonging to the groups $r = 0, 1, 2$, and 3. At the start of the pumping, prior to the onset of stimulated emission of photons with the energies $\hbar\varpi_{e,-1} = \hbar\varpi_{a,-2}$ the populations of the $r = 2$ and $r = 3$ groups too can be neglected. Then, in the case of moderate pump intensity with spectral photon flux density j_p satisfying the inequality

$$j_p << j_h \equiv \frac{1+\alpha}{\lambda^2}\frac{2J_1+1}{2J_2+1} \tag{53}$$

the density of the excited nuclei builds up to the point in time t_p from the onset of irradiation, when $n_2 = 0$ and $n_1 = n_{10}$, as

$$n_2 \approx n_{10}\frac{j_p}{j_h}\left[1-\exp\left(-\frac{t_p}{\tau}\right)\right] \tag{54}$$

Here,

$$\tau = \frac{2\pi\hbar}{\Gamma_\gamma(1+\alpha)} \tag{55}$$

is the lifetime of the level 2 and α is the coefficient of the internal conversion. For instance, $n_2 = 10^8\,\mathrm{cm}^{-3}$ when $E_0 = 10$ keV, $j_h = 10^{16}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{Hz}^{-1}$, $n_{10} = 10^{14}\,\mathrm{cm}^{-3}$, and $j_p = 10^{10}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{Hz}^{-1}$.

We emphasise that the density of the $r = 1$ nuclear group given by formula (54), i.e. the population density of the laser level, is in effect independent of the radiative width Γ_γ. This opens up the possibility of employing nuclei with arbitrary, including fairly large, radiative widths that do not require excessively deep «cooling» of the nuclear ensemble.

An important advantage of the «two-level» scheme is that it offers the freedom to choose a nucleus from the isotope table, since this scheme does not impose requirements on the third level of a candidate. This freedom is gained through drawing the energy of stimulated emission exclusively from an incoherent X-ray pump source. The action of a «two-level» pumping scheme can be treated as the anisotropic quantum amplification in stimulated backward resonance nuclear scattering. A very important point is that the anisotropy (the unidirectional character of the amplified photon flux) is attained without the use of mirrors.

When the beam of «cooled» nuclei and the photon pump beam are precisely collinear, the intensity of the latter may decrease substantially owing to the beneficial absorption by the nuclei under excitation. This limits both the amplification length $L < \left(\sigma_p n_{10}\right)^{-1}$ and the total number of excited nuclei N_t over a unit beam cross-section of length L $(N_t < \sigma_p)$, which in essence determines the overall gain - length product (here σ_p is the cross-section of the pump transition).

To obviate these limitations, it would suffice to introduce a slight departure from the collinear propagation by a small angle $\phi > \sigma_p n_{10} D$, where D is the lateral dimension of the nuclear stream. Since this dimension can be asssumed to be sufficiently small (see section 6), the limitations fail for $\phi << \pi$ (for instance, the length L is unlimited for $\phi \approx 15$ mrad, D = 0.015 cm, and $\sigma_p n_{10} = 1$ cm^{-1} opposed to $L < 1$ cm for exact collinearity).

9. Anti-Stokes pump scheme

The anti-Stokes pump scheme [34 - 41] is commonly referred to as the trigger scheme (Fig. 5). A photon from an external source with energy $\hbar\varpi_p \approx E_2 - E_3 = E_p$ absorbed by a nucleus liberates the energy stored in the relatively longlived metastable state 3 instead of delivering the energy for subsequent lasing, and circumvents the selection rules which maintain its population density. Therefore, this scheme offers advantages over both the standard three-level and the «two-level» schemes, for in this case the relatively long-lived metastable states of isomeric nuclei are used as the primary energy supplier. This scheme offers even higher energy efficiency, since the ratio between the laser photon energy $\hbar\varpi \approx E_2 - E_1 = E_0$ and the pump photon energy $\hbar\varpi_p$ may be substantially greater than unity. The feasibility of anti-Stokes process in long-lived isomers was verified experimentally in Refs [42 - 45].

In this scheme the energies of the laser and pump photons are significantly different. Therefore, in addition to expressions (31) and (32) which can be referred to laser photons here, account must be taken of the expression for the recoil energy

$$E_{rp} = \frac{\left(\hbar\varpi_p\right)^2}{2Mc^2} = \frac{E_p^2}{2Mc^2} \qquad (56)$$

of the corresponding nuclear velocity increment

$$\Delta v_p = 2c\frac{E_{rp}^2}{E_p} \qquad (57)$$

and of the Doppler shift

$$\hbar\varpi_p \frac{\Delta v_p}{c} = 2E_{rp} \qquad (58)$$

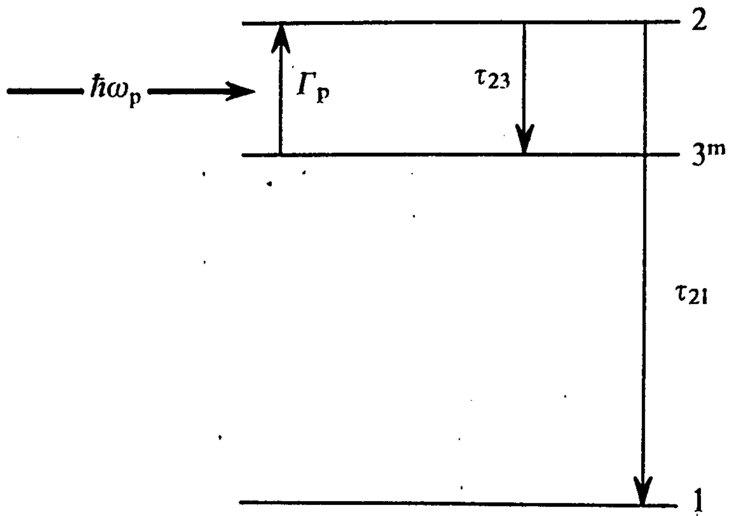

Figure 5. Anti-Stokes pumping scheme involving absorption of a triggering photon with the energy $\hbar\omega_p$ and transition from the metastable 3^m level to level 2 (Γ_p and τ_{23} are the radiative width and the lifetime of the $2 \rightarrow 3$ transition, τ_{21} is the spontaneous lifetime of the $2 \rightarrow 1$ laser transition).

which are due to the 3 $\rightarrow$ 2 transitions in the absorption or the emission of photons with the energy $\hbar\varpi_p \approx E_p$.

Subsequent consideration follows the chain of argument employed in section 7. Initially, metastable isomers in state 3 and unexcited nuclei in state 1 are present in the nuclear ensemble r = 0 with velocity $\Delta v =$ 0. Absorption of a pump photon with energy $\hbar\varpi_p = E_p + E_{rp}$ gives rise to the new r = 1 nuclear group in state 2 with the velocity $\Delta v_1 = \Delta v_p$. The inverse 2 $\rightarrow$ 3 transitions under pump irradiation introduce nothing essentially new as compared with the «two-level» scheme: the tendency for saturation of the transition under pumping shows up, as does the possibility of hidden inversion for the pump of photons with energy

$$\hbar\varpi_{e,-1} \approx E_p - 3E_{rp} \tag{59}$$

propagating in the -z-direction. The emission of these photons gives rise to the new r = 2 group of metastable nuclei with the velocity $\Delta v_2 = 2\Delta v_p$.

Of greater significance are the 2 $\rightarrow$ 1 transitions in the nuclei of the r = 1 group with emission of photons with energy

$$\hbar\varpi_{e,\pm 1} \approx E_0 - E_{rec} \pm 2E_o \frac{E_{rp}}{E_p} \tag{60}$$

(the signs refer respectively to the $\pm$ z-directions of emission). Here $E_0 = E_2 - E_1$, second term accounts for the contribution of recoil and the third for the Doppler shift owing to the velocity increment Δv_p. These transitions are responsible for the creation of a new group of nuclei in state 1 with the velocity

$$\Delta v_{\pm 3} = \Delta v_0 + \Delta v_p \mp \Delta v \approx 2c\frac{E_{rec}}{E_p}\left(\frac{E_p}{E_0} \mp 1\right) \tag{61}$$

The subsequent possible stages of the radiative transition cascade need not be considered for the same reasons as in section 8.

The absorption resonance in the 1 $\rightarrow$ 2 transition in the nuclei of the initial r = 0 group corresponds to the photon energy $\hbar\varpi_{a0} = \hbar\varpi_0$ [Eqn (9)] of either propagation direction, and hidden inversion occurs between states 2 and 1, since the emission and absorption lines for the $\pm$ z-directions are shifted in energy by

$$\hbar\varpi_{a0} - \hbar\varpi_{e,\pm 1} = 2E_{rec} \mp 2E_0 \frac{E_{rp}}{E_p} = 2E_{rec}\left(1 \mp \frac{E_p}{E_0}\right) \tag{62}$$

The pump kinetics is governed by rate equations in which the terms representing the spontaneous emission of relatively long-lived initial metastable isomers can be neglected. If the

spectral density of the pump photon flux j_p is far less than the parameter of the 3 $\rightarrow$ 2 trigger transition,

$$j_p << j_a \equiv \frac{1+\alpha_{23}}{\lambda_p^2} \frac{2J_3+1}{2J_2+1} \frac{\tau_{23}}{\tau_2} \qquad (63)$$

by time t_p the density of nuclei n_2 in level 2 of the laser transition builds up to

$$n_2 \approx n_{30} \frac{j_p}{j_a} \left[1 - \exp\left(-\frac{t_p}{\tau_2} \right) \right] \qquad (64)$$

Here λ_p is the pump wavelength, J_2 and J_3 are the angular momenta of states 2 and 3, α_{23} is the coefficient of internal electron conversion of the 2 $\rightarrow$ 3 transition, n_{30} is the initial density of the isomeric nuclei in metastable state 3,

$$\tau_2^{-1} = \tau_{21}^{-1} + \tau_{23}^{-1} \qquad (65)$$

is the total decay probability of state 2, τ_{21} and τ_{23} are the total lifetimes of the 2 $\rightarrow$ 1 and 2 $\rightarrow$ 3 laser transitions. For instance, $n_2 \approx 10^9$ cm^{-3}, E_p = 1 keV, $j_a = 10^{15}$ cm^{-3} s^{-1} Hz^{-1}, n_{30} = 10^{13} cm^{-3}, and $j_p = 10^{11}$ cm^{-2} s^{-1} Hz^{-1}.

10. Photon losses in the nuclear medium. Techniques for their reduction and modulation

The total cross-section κ of photon losses due to the photoelectric effect and Compton scattering , which enters in the coefficient K [Eqn (15)], is generally far less than the cross-section of stimulated emission σ_0 [Eqn (2)] for not-too-small linewidth ratios β [Eqn (3)]. For instance, $\sigma_0 = 2.5 \times 10^{-17}$ cm^2 as compared with $\kappa \approx 5 \times 10^{-22}$ cm^2 for aluminium and $\kappa \approx 10^{-20}$ cm^2 for lead ($\hbar\varpi =$ 10 keV).

Nevertheless, under the conditions of hidden inversion when stimulated emission can proceed for $n >> n_2$ too (see section 4), it is not improbable that $g = \sigma_0 n_2$ proves to be comparable with the product κn and the coefficient K [Eqn (15)] small or even negative if $\kappa / \sigma_0 > n_2 / n$

Hence it is necessary to reduce the photon losses by removing excess atoms not bearing excited nuclei from the amplification channel. We emphasise that this step is taken to reduce the photon losses due to atomic electrons and, consequently, to lower the amplification threshold rather than to prepare an excess of the number of excited nuclei over the number of unexcited ones, which is not required in the case of hidden inversion.

The nuclear recoil produced under X-ray photon pump helps in the solution of the problem [33]. As follows from section 7, the absorption of pump photons gives rise to two kinematic

nuclear groups -unexcited ($r = 0$) and excited ($r = 1$)- which differ in velocity by $\Delta v = 2cE_{rec}/E_0$. Consequently, it also gives rise to the kinematic Doppler shift

$$\Delta \varpi_{at} = \varpi_{at} \frac{\Delta v}{c} = 2\varpi_{at} \frac{E_{rec}}{E_0} \tag{66}$$

of the electron transitions in the corresponding atoms (ϖ_{at} is the electron transition frequency).

A typical relative shift of atomic resonances $\Delta\varpi_{at}/\varpi_{at} \approx 10^{-6} - 10^{-7}$, which significantly exceeds the Doppler broadening of the lines of the electron transitions in a «cooled» atomic ensemble. This shift can be used selectively to ionise atoms in the $r = 0$ group by the radiation of an optical laser and to remove the resulting positive ions from the amplification channel by employing an external lateral electric field (Fig. 6). Today, the similar technique of selective multistage photoionisation [46 - 49] has been safely embodied, in particular, in techniques of laser isotope separation [50, 51]. This technique enables the undesirable impurity densities to be reduced by several orders of magnitude.

Besides, the use of this technique makes it possible to do more than selectively remove from the amplification channel the atoms responsible for excess photon losses. It also provides a possibility to implement a process similar to Q-switching in lasers operating in the optical range to generate the so-called giant light pulse. Excited nuclei are stored under pumping over the interval $0 \le t \le t_p$ [Eqn (54)]. Initially, all the nuclei are unexcited. The storage proceeds for relatively high photon losses, i.e. well below the threshold of stimulated emission (Fig. 7). Then, at $t \approx t_p$ [Eqn (54)], a pulsed removal of the atoms bearing unexcited nuclei is achieved by selective laser photoionisation to decrease sharply the amplification threshold. This is accomapnied by pulsed stimulated emission of gamma-photons, much like the giant-pulse emission in an optical laser.

11. Laser cooling and confinement of neutral nucleus-bearing atoms

In the previous sections we recognised the necessity of preparing a monokinetic ensemble of nuclei as a means of increasing the β ratio. Therefore we revert to the problem of deep cooling an ensemble of nucleus-bearing atoms. Nowadays the light pressure of a laser beam has become an efficient instrument in physical experiments on cooling and manipulation of ensembles of neutral atoms (see, e.g. Ref. [32]).

As is known, the action of a directional laser beam on a neutral atom involves multiple recurrence of a cycle consisting of resonance absorption of a photon with energy $\hbar\varpi_L$, wave vector $\boldsymbol{k}$, and momentum $\hbar\boldsymbol{k}$ ($\boldsymbol{k}$ and $\hbar\boldsymbol{k}$ are fixed in direction) followed by spontaneous emission of a photon with the same energy but arbitrarily directed wave vector. After averaging over many emission events the momentum transferred to the atom proves to be close to zero owing to isotropy of the spontaneous emission. The total momentum transferred to the atom after N_a absorption events coincides in direction with the laser beam and equals $N_a\hbar\boldsymbol{k}$. The minimum duration of the absorption - emission cycle, which determines the maximum number of cycles per unit time, for a laser beam of sufficiently high intensity

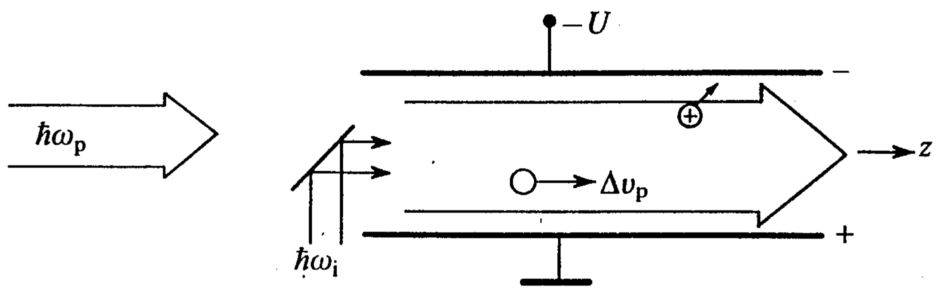

Figure 6. Use of selective photoionisation technique for freeing the amplification channel from the atoms responsible for excess photon losses (○ – neutral atoms with excited nuclei and velocity increment Δv_p, ⊕ — positive ions with unexcited nuclei, $\hbar\omega_p$ — pump radiation, $\hbar\omega_i$ — ionising radiation of an optical laser, $-U$ — voltage across the electrodes).

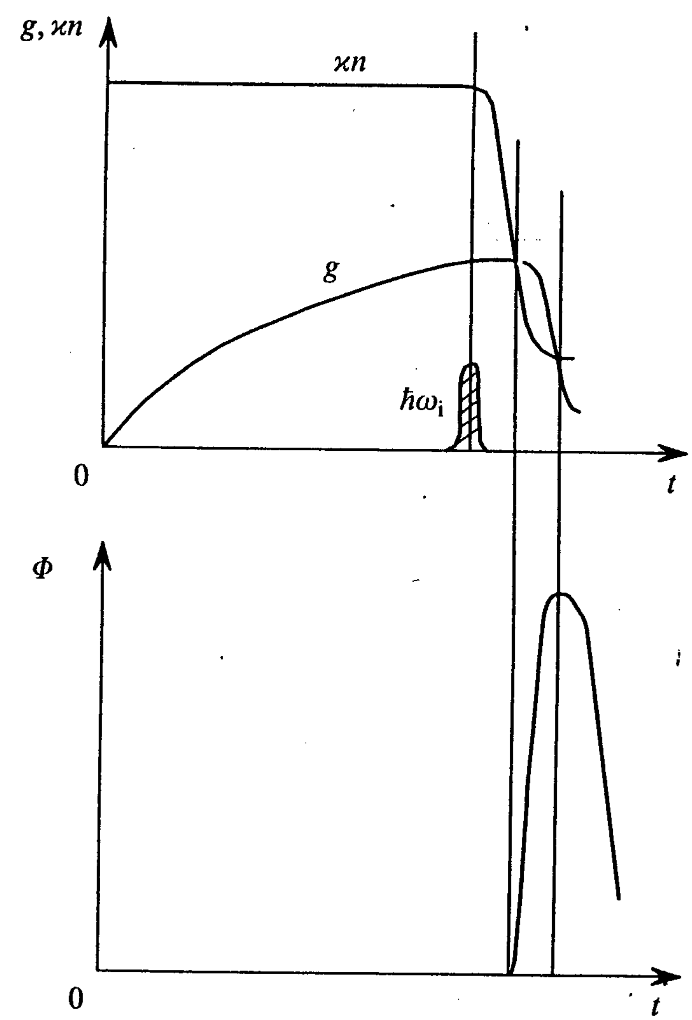

Figure 7. Stimulated emission of a γ-ray photon pulse through the modulation of losses due to the photoelectric effect from atomic electrons, the Compton scattering, etc. ($g = \sigma_0 n_2$ – gain coefficient, $\varkappa n$ – coefficient of photon losses (extinction coefficient), $\hbar\omega_i$ – ionising pulse of an optical laser, Φ – stimulated γ-ray radiation pulse).

$j >> (\sigma_{at}\tau_{at})^{-1}$ is $\Delta t_{at} \approx 2\tau_{at}$, where σ_{at} is the resonance absorption cross-section and τ_{at} is the spontaneous relaxation time. Hence the number of cycles N_a required for directional transfer of the momentum corresponding by the order of magnitude to the atomic thermal velocity ~ 10^5 cm s^{-1} is approximately 10^5 for $|\hbar k| \approx 10^{-22}$ g cm s^{-1}. The total period of time $N_a \Delta t_{at}$ for the operation is therefore rather long - of the order of milliseconds. As applied to the problem of a nuclear gamma-ray laser with relatively fast laser transitions, this implies the following. Unless alternative cooling methods are applied to decrease radically the operation time, the laser cooling of an atomic ensemble should be achieved at the stage preceding the stage of excitation (pumping) of nuclei, as noted above.

Various modifications of the general approach discussed above, including quite sophisticated ones, enable the laser cooling of atoms down to different limiting temperatures marked by the reference points in section 5. The abbreviations used there are interpreted as follows [32]: DC, Doppler cooling (the so-called optical molasses in the 3-D case); SDC, sub-Doppler cooling (including the «Sisyphean effect»); SRC, sub-recoil cooling; EC, evaporative cooling.

As adopted in Section 6, the optimum shape of the amplifying medium in the primary gamma-ray lasing experiment is an extended straight filamentary beam of nucleus-bearing atoms, which measures fractions of a millimetre in diameter. Therefore steps should be taken to ensure lateral confinement of the atomic beam throughout the length of the amplification channel, in addition to longitudinal monokinetisation ensuring an acceptable linewidth ratio β. A problem of this kind is solved successfully by combined magneto-optical techniques and by purely laser techniques [32].

Techniques of the former type are exemplified by the so-called «funnel» for atoms (Fig. 8) [32]. This is a system of four parallel conductors arranged at the corners of a square. The currents of each set of two neighbouring conductors are opposite in sign to induce a quadrupole magnetic field in the vicinity of the system axis. The light pressure of two pairs of oppositely directed laser beams orthogonal to the axis cools the atoms and focuses them in the paraxial region of the magnetic trap. Another two oppositely directed laser beams collinear with the axis are tuned away from the resonance frequency of the atomic transition (the detunings are opposite in sign) and make the atoms monokinetic relative to a nonzero longitudinal velocity to form the so-called moving optical molasses. In one of the first experiments [52], the method described above was used to obtain a beamof sodium atoms with a translational velocity of 1 - 5 m s^{-1}, a temperature of about 0.2 mK, and a density of the order of 5×10^8 cm^{-3}.

Purely laser methods include optical dipole traps [32] in which atoms previously cooled to millikelvin temperatures become concentrated either along the axis of an intense laser beam, with frequency detuned down relative to the resonance frequency, or along a beam with a ring-shaped cross section and intensity minimum on the axis, and detuned up from the resonance.

Of course, these and other methods of forming extended monokinetic atomic beams [32] are by no means a complete solution to the problem of preparation of cooled free nuclear ensembles for a gamma-ray laser but serve as a safe experimental basis. The main problem here is possibly the necessity of selecting a candidate isotope which combines the nuclear

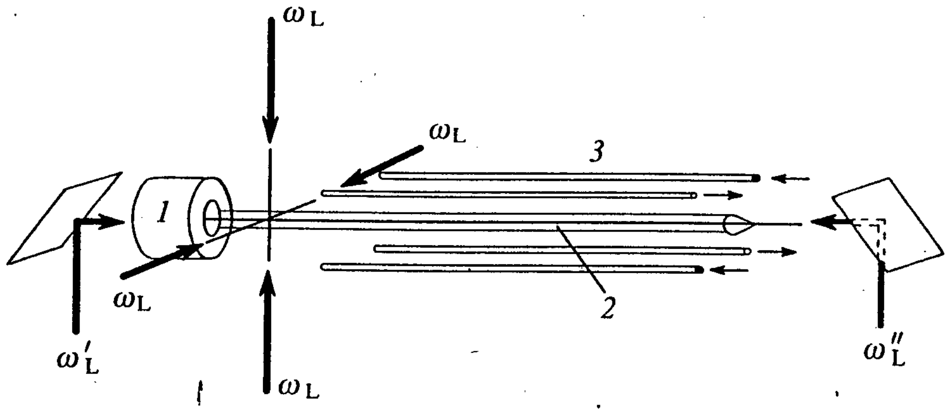

Figure 8. 'Funnel' for cooling and confining atomic beams: (*1*) atomic source; (*2*) atomic beam; (*3*) current-carrying conductors inducing a quadrupole magnetic field; lateral (ω_L) and cooling (ω'_L and ω''_L) laser beams.

properties suited for efficient quantum amplification with the atomic properties suited for the preparation of the requisite cooled atomic beam.

12. Electrokinetic preparation of a monokinetic nucleus-bearing ion beam

This method of preparing longitudinally monokinetic beams of charged particles has long been employed to prepare low-noise electron beams for use in microwave electronics and represents the simplest method. Essentially the method is as follows [24, 27]. The scatter in the longitudinal velocities of charged particles decreases sharply as they are accelerated by the electric field owing to the quadratic kinetic energy - velocity dependence (Fig. 9). Since the Doppler broadening is proportional to the scatter of velocities and not energies, the new effective «temperature» can be defined as [24]

$$T_{=} = \frac{\ln 2}{4} \frac{kT_0^2}{eU} \qquad (67)$$

where eU is the ion energy on acceleration and T_0 is the initial temperature. For instance, the acceleration to 100 keV reduces the «temperature» from $T_0 = 80$ K to $T_{=} = 1\ \mu$K. To avoid a decrease in ion beam density accompanying the acceleration by a constant field, pulsed acceleration should be applied to all ions of a selected group simultaneously confined in the electrode spacing.

Expression (67) was obtained in the purely kinematic approximation, neglecting the Coulomb ion - ion interaction. Needless to say, the inclusion of perturbations caused by beam space-charge effects and various plasma phenomena imposes serious limitations on the ion beam density and casts doubt on the prospects of the electrokinetic «cooling» in the implementation of a nuclear gamma-ray laser [24, 27].

13. X-ray pump sources

Without going into the presentation of the physical processes in X-ray radiation sources of various types, we present here a brief list of the main possible candidates for the role of an «optical» pump for «cooled» free-nucleus ensembles.

Nowadays classical bremsstrahlung sources are capable of delivering huge integral X-ray photon fluxes in a broad spectral range up to tens of megaelectron volts [53]. However, they do not offer the spectral density required to achieve effective pump.

Different types of free-electron radiation could be among the most convenient for use under the intended experimental conditions. They offer such beneficial properties as a well-shaped directional X-ray photon beam and a quasi-continuous radiation mode. Regrettably, the spectral radiation density of classical synchrotron sources with a circular orbit of relativistic electrons is moderately high. An elementary estimate of its magnitude can be obtained by employing the formula for calculating the irradiance (in W mm^{-2}) [54]:

$$I_{\lambda} = 59 \frac{E^2 I}{\lambda l^2} \frac{\eta_1}{\eta_2} \frac{\Delta\varpi}{\varpi} \qquad (68)$$

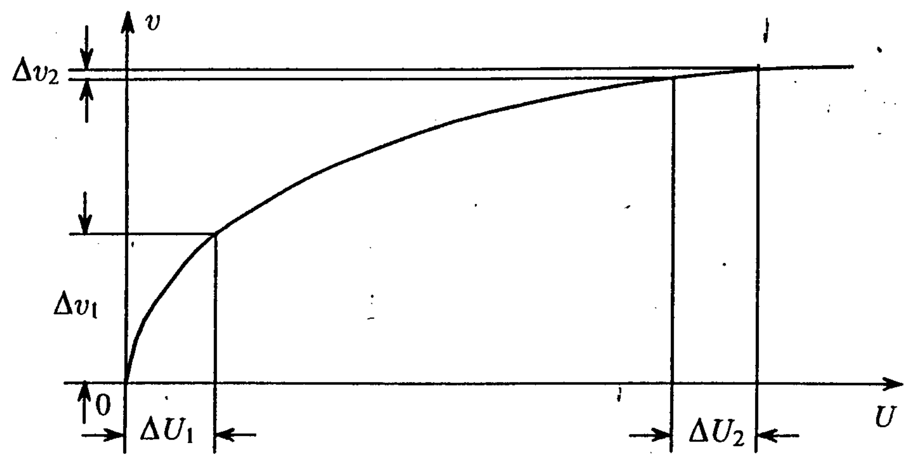

Figure 9. Square-root dependence of the ion velocities v on the accelerating voltage U which illustrates a reduction in the ion velocity scatter for an invariable scatter in energy on acceleration.

where E is the electron energy in GeV; I is the beam current in amperes; λ is the wavelength in Angstroms, l is the distance to the electron orbit in metres; $\Delta\varpi/\varpi$ is the relative frequency band; η_1 and η_2 are the universal functions of the ratio between the wavelength λ and the critical wavelength (in Angstroms)

$$\lambda_c = 5.59\frac{R}{E^3} \tag{69}$$

represented graphically in Ref. [54]; R is the orbit radius in metres. From expression (68) for the irradiance follows an estimate of the sought-for spectral density j_p (in photons per cm^2 s Hz) entering in formulas (54) and (64):

$$j_p = \frac{I_\lambda}{\hbar\varpi_p} = E^2\frac{I\lambda}{l^2}\frac{\eta_1}{\eta_2} \tag{70}$$

[here the frequency band $\Delta\varpi/2\pi$ in formula (68) is taken to be 1 Hz]. It proves to be unacceptably low, not exceeding hundreds of photons per cm^2 s Hz.

A notable gain, conceivably by many orders of magnitude, can be obtained by using wigglers that select one radiation harmonic [54], free-electron lasers [55, 56], relativistic Doppler up-conversion in the scattering of low-frequency radiation by relativistic electrons [57, 58], and may also be obtained by using the Doppler conversion in the resonance scattering by electron oscillators [59]. Certain expectations are related to the information on the recent launch of an X-ray radiation source (the Advanced Photon Source) in the Argonne National Laboratory, USA. As reported in Ref. [60], the source brightness exceeds all previously attained values by a factor of 10 000. Expectations have also been raised by the development of an X-ray free-electron laser with a ten-billion-fold increase in brightness, which is projected for 2005 (SLAC, Stanford, USA) [61]. Extensive research programmes for the development and the application of high-brightness X-ray sources are also being pursued in Japan (Advanced Photon Research Center, Kyoto) [62], Germany [63], and other countries.

Another source of intense X-ray radiation is a dense high-temperature plasma. If Planck's formula for blackbody radiation is used to obtain a crude estimate of the spectral density of plasma radiation, the temperature required to attain the spectral density j_p is given by

$$kT_p = \hbar\varpi_p \ln^{-1}\left(\frac{2\pi}{\lambda^2 j_p} + 1\right) \tag{71}$$

Hence it follows that $j_p = 10^8$ photons per cm^2 s Hz can be obtained for a temperature Tp of about 10 eV for $\hbar\varpi_p = 0.1$ keV, about 70 eV for $\hbar\varpi_p = 1$ keV, and about 500 eV for $\hbar\varpi_p = 10$ keV.

Plasmas with the specified temperatures are produced, in particular, at the focus of terawatt optical lasers with condensed targets of special design (see, e.g. Ref. [64]). Extremaly efficient energy conversion into X-ray radiation is achieved in Z-pinch plasma experiments (with or

without «hohlraum») at Sandia National Laboratory, Los Alamos National Laboratory, Lawrence Livermore National Laboratory, TRINITY (Russia) and other facilities [89-92]. Also, intense discrete X-ray lines are generated with a relatively high conversion coefficient (over 10^{-5}) [65]. Needless to say, in the latter case it is necessary to ensure a coincidence of the X-ray lines with the nuclear transitions to be pumped [66, 67].

The spectral density of the isotropic emission of a plasma volume with diameter d decreases sharply with distance to the object l as

$$j_p = j_p(d)\left(\frac{d}{2l}\right)^2 \tag{72}$$

Therefore, care must be taken to transport and focus the plasma emission to the nuclear beam, which is a serious problem in the X-ray range. In its long-wavelength domain it is possible to employ multilayer mirrors with reflectivities up to several tens percent (see, e.g. Ref. [68]). It is conceivable that focusing hollow multimode waveguides [69,70] could also be employed if good use is made of total external reflection to extend them from the visible to the X-ray range [71]. Such waveguides would be capable of transferring the focused image of a plasma volume for distances of tens of centimetres, simultaneously acting as absorption filters to reject undesirable frequencies owing to their dispersion properties.

We know that the spectral density of the plasma X-ray lasers, which can be estimated at 10^{13} - 10^{15} photons per cm^2 s Hz 1 [4], is sufficient for pumping the nuclear ensembles. Researchers from the Lawrence Livermore National Laboratory [72] estimate the effective Planck's temperature of the X-ray laser radiation at 1 GeV. If we adopt this estimate, the spectral density calculated from formula (71) can become very high - of the order of 10^{20} photons per cm^2 s Hz - in discrete lines in the 35 - 3.5 nm interval. The representative wavelengths are as follows: 15.5 nm for Yt [72], 14.0 nm for Ag, 12.6 nm for In, 12.0 nm for Sn, 7.3 nm for Sm [73], 46.9 nm for Ar, and 60.8 nm for S [74], etc.

The virtue of X-ray lasers is that they produce a high intensity well-shaped directional photon beam. Among their disadvantages are: short pulse durations (below 1 ns), difficulties encountered in extending lasing to shorter wavelengths, and the necessity of making X-ray lines agree with the nuclear ones [66, 57].

Finally it should be admitted that the question of the optimum x-ray source for pumping the nuclear ensembles is still an open question.

14. Layout of the primary experiment

By way of demonstrating the feasibility of the primary experiment, Fig. 10 shows a configuration synthesising the partial solutions discussed in the previous sections.

First and foremost, the configuration comprises a device for cooling, forming, and confining a beam of nucleus-bearing atoms of a type described in section 11. A version of this device (a «funnel») without details is shown in Fig. 8 by dashed lines. It includes an atomic source (1), light beams of cooling and collimating lasers ϖ_L, straight current-carrying conductors to induce a quadrupole magnetic field of a magneto-optical trap (2), etc. Combined with the trap are the electrodes with a voltage drop $-U$ across them. The electrodes are intended for the

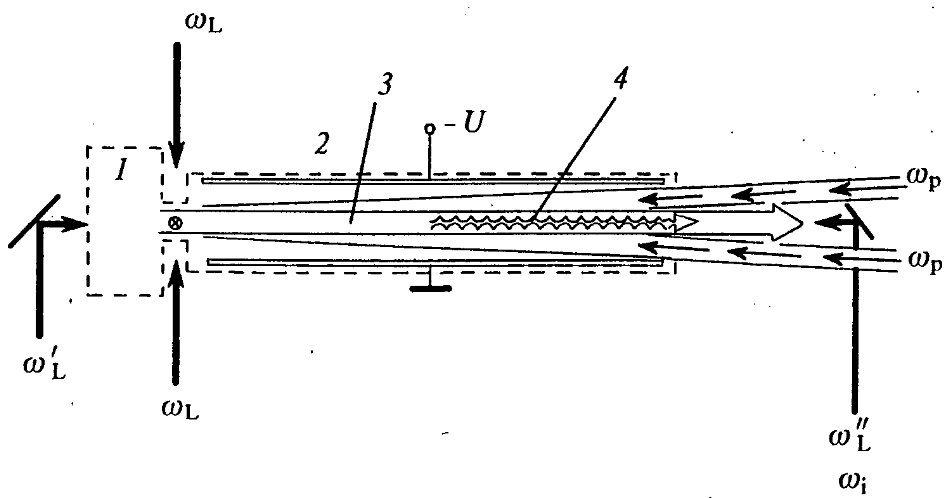

Figure 10. Graphic representation of the primary experiment to observe quantum amplification of γ-ray radiation in a cooled nuclear medium with hidden inversion: (*1*) source of nucleus-bearing atoms, (*2*) atomic trap, (*3*) beam of cooled nuclei, (*4*) beam of stimulated γ-ray radiation, transverse (ω_L) and cooling (ω'_L and ω''_L) laser beams, ionising laser radiation (ω_i), cleansing voltage ($-U$), x-ray pump photons (ω_p).

photoionisation cleansing of the atomic beam from the atoms bearing unexcited nuclei whose electrons are responsible for excess gamma-photon losses and also for the modulation of these losses (the ionising laser beam is designated ϖ_i; see also section 10 and Figs 6 and 7). A cooled atomic beam (3) propagates along the trap axis from left to right, in opposition to a pump X-ray photon stream ϖ_p, which is directed at a small angle relative to the beam axis. The gamma-photon flux, which has experienced quantum amplification, is represented by a wavy line (4) with an arrow pointing to the right. Needless to say, the experiment layout given here is nothing more than a sketch, and not the only one possible, devoid of the details and serving merely to illustrate the basic concept.

It is of interest to estimate the instantaneous value of the directional monochromatic output flux of gamma-photons in the context of the simplified stationary model of section 5 employing expression (24) for the quantum efficiency Q and preassigning the total number of excited nuclei N in the amplification channel:

$$I = QN/\tau_\gamma \tag{73}$$

For instance: $I = 10^{11}$ s^{-1} for $N = 10^{12}$, $\tau_\gamma = 1\,\mathrm{ns}$, $G = 2.5$, $D = 0.1$ mm, $L = 5$ m, and $\beta = 0.9$.

15. Examples of potentially useful isotopes

Regrettably, the paucity of available data on the low-energy radiative nuclear transitions (first and foremost on the life-times and on the internal conversion coefficients) hinders reliable selection of appropriate candidate isotopes and especially the optimisation of the choice. Many researchers, including those at the Los Alamos National Laboratory, have pointed out repeatedly that the required information is scarce [75]. This scarcity is clearly demonstrated by Tables 1 and 2, compiled primarily employing the data bank of Ref. [76], with question marks in place of the missing or unreliable data. Therefore, the selection of examples in Tables 1 and 2 is rather arbitrary, serves as an illustration, and by no means is an attempt of optimisation. The latter task is of exceptional importance and calls for both organising information retrieval from all existing data banks and, possibly, additional nuclear spectroscopic studies. Moreover, as indicated in section 11 it is imperative that the atomic properties of the candidates should also be taken into consideration to ensure efficient preparation of the cooled nuclear ensembles of extended filamentary shape.

The paucity of information and the disinclination to ascribe assumed characteristics to the existing isotopes force us to resort to an artificial procedure to construct a plausible model which could serve as a basis for the quantitative estimates of the conditions for possible experiments. To this end, the last two lines of Tables 1 and 2 give the data on the two hypothetical isotopes $^{200}X_h$ and $^{200}X_a$ with arbitrarily adopted but quite realistic parameters, and Tables 3 and 4 present the results of the corresponding calculations of the main laser characteristics.

The data of Table 3 draw attention to the feasibility of reliable observation of anisotropic quantum amplification in the «two-level» scheme, with the G criterion (22) notably exceeding unity. This is primarily due to the existence of a series of isotopes with suitable transition

Table 1. 'Two-level' scheme (examples).

Isotope	$E_0 = E_2 - E_1/\text{keV}$	J_2	J_1	τ/ns	α
$^{134}Cs_{55}$	11.24	5^+	4^+	46.6	?
$^{141}Sm_{62}$	1.58	$3/2^+$	$1/2^+$	?	?
$^{169}Tm_{69}$	8.41	$3/2^+$	$1/2^+$	4.08	?
$^{183}W_{74}$	46.48	$3/2^-$	$1/2^-$	0.188	?
$^{193}Pt_{78}$	1.642	$3/2^-$	$1/2^-$	9.7	?
$^{201}Hg_{80}$	1.56	$1/2^-$	$3/2^-$	?	?
$^{200}X_h$	1	$3/2^+$	$1/2^-$	1	<1
	10	$3/2^+$	$1/2^-$	1	<1

Table 2. Anti-Stokes scheme (examples).

Isotope	$E_0 = E_2 - E_1/\mathrm{keV}$	$E_p = E_2 - E_3/\mathrm{keV}$	J_2	J_3	J_1	τ_{21}/ns	τ_{23}/ns	τ_{31}	α_{21}	α_{23}
$^{73}Se_{34}$	26.32	0.61	?	$3/2^-$	$9/2^+$	?	?	39.8 min	?	?
$^{96}Tc_{43}$	35.4	1.12	?	4^+	7^+	?	?	51.5 min	?	?
$^{150}Eu_{63}$	43	0.9	?	0^-	?	?	?	12.8 h	?	?
$^{179}Hf_{72}$	1105.9	0.077	?	$25/2^-$	$9/2^+$	?	?	25.05 d	?	?
$^{242}Am_{95}$	52.9	2.27	?	5^-	1^-	?	?	141 yr	?	?
$^{200}X_a$	10	0.5	$3/2^+$	$1/2^-$	$5/2^+$	1	1	>1 h	<1	<1
	20	1.0	$3/2^+$	$1/2^-$	$5/2^+$	1	1	>1 h	<1	<1

Table 3. Two-level' scheme (calculations).

Parameter	Formula	$^{200}X_h$	$^{200}X_h$
β	–	0.95	0.95
$T_{\parallel}$/mK	(26)	7.6	0.76
j_h/cm^{-2} s^{-1} Hz^{-1}	(53)	3.2×10^{13}	3.2×10^{15}
j_p/cm^{-2} s^{-1} Hz^{-1}	–	10^{12}	2×10^{14}
kT_p/eV	(71)	167	1900
t_p/ns	–	3	3
n_2/n_{10}	(54)	0.03	0.06
T_h/mK	(10)	8.8	1100
n_{10}/cm^{-3}	–	5×10^{13}	5×10^{14}
n_2/cm^{-3}	(54)	1.5×10^{12}	3×10^{13}
g/cm^{-1}	(12)	0.0035	0.0007
L/cm	–	500	500
gL	–	1.75	0.352
G	(22)	2.7	1.2
$\varkappa$/cm^2	–	10^{-18}	10^{-18}
D/cm	(27)	0.1	0.01
V/cm^3	–	3.9	0.039
N	–	5.9×10^{12}	1.2×10^{12}
I/s^{-1}	(73)	3.8×10^{13}	3.4×10^{10}

Table 4. Anti-Stokes scheme (calculations).

Parameter	Formula	$^{200}X_a$	$^{200}X_a$
β	–	0.95	0.95
$T_{\|\|}$/mK	(26)	0.75	0.375
j_a/cm^{-2} s^{-1} Hz^{-1}	(63)	1.6×10^{13}	6.5×10^{13}
j_p/cm^{-2} s^{-1} Hz^{-1}	–	3×10^{12}	10^{13}
kT_p/eV	(71)	143	270
t_p/ns	–	1.5	2.3
n_2/n_{30}	(64)	0.18	0.18
T_h/mK	(10)	1770	7200
n_{30}/cm^{-3}	–	5×10^{14}	2×10^{15}
n_2/cm^{-3}	(64)	0.9×10^{14}	2.8×10^{14}
g/cm^{-1}	(12)	0.00204	0.0016
L/cm	–	500	500
gL	–	1.07	0.8
G	(22)	1.8	1.54
$\varkappa$/cm^2	–	10^{-18}	10^{-18}
D/cm	(27)	0.01	0.003
V/cm^3	–	0.04	0.0035
N	–	3.5×10^{12}	10^{12}
I/s^{-1}	(73)	1.5×10^{11}	3.3×10^9

energies ($E_0 \approx 1-10$ keV) and other parameters (as far as they are known, see Table 1). Calculations employing the hypothetical data on «lucky» isotopes (Table 3) suggest that the required temperatures $T_=$ (26) of cooled atomic ensembles (of the order of 1 mK) are attainable. These temperatures ensure simultaneously a high ratio $\beta \to 1$ (3) and the existence of hidden inversion [$T_= < T_h$ (10)] for a small relative density of excited nuclei n_2/n_{10} (54) of about several percent.

Much more rigorous are the requirements imposed on the spectral density of the pump radiation with effective black-body temperature T_p of the order of 0.1 - 1 keV, as well as on the initial nuclear density n_{10} in the filamentary amplification channel (about 10^{14} cm^{-3}). However, the total number of excited nuclei is moderate ($N \approx 10^{12}$). As regards the combination of experimental parameters, lower-energy transitions have many points in their favour, all other factors being equal.

Qualitatively similar conclusions can be drawn in the consideration of Tables 2 and 4. The former exemplifies several isomers supposedly suitable for conducting experiments on the quantum gamma-ray amplification under anti-Stokes pump (Fig. 5). While the scatter in energy E_0 of the proposed laser transitions is wide (from tens to thousands of kiloelectronvolts), the energies E_p of the triggering pump quanta are all close to 1 keV (sometimes even to 100 eV) to furnish very high energy efficiency: from several tens to tens of thousands. In this case, the lifetimes τ_{31} of the initial metastable isomeric state range from several hours to hundreds of years.

Table 4 lists the calculated parameters of the experiment on the hypothetical «lucky» nuclei. Noteworthy is the fact that the criterion for the observation of quantum amplification with the gain G [Eqn (22)] exceeding unity is realised when the nuclear ensemble is cooled to moderately low temperatures $T_=$ [Eqn (26)], of the order of tenths of a millikelvin. On the strength of the inequality $T_= < T_h$ [Eqn (10)], they simultaneously predetermine the existence of hidden inversion for a relative nuclear density n_2/n_{30} [Eqn (64)], of about ten percent, of the upper level of the laser transition.

As in the «two-level» version of the experiment, the effective blackbody temperature T_p characterising the required spectral density of the pump radiation amounts to hundreds of electron volts. In this case the initial density n_{30} of the metastable isomer is rather high (10^{14} - 10^{15} cm^{-3}) for a moderate total number of nuclei N (of the order of 10^{12}) in the upper level of the laser transition. Here the advantage of nuclei with relatively low-energy transitions too is evident.

The instantaneous directional monochromatic output flux of gamma-photons I estimated by formula (73) is tangible for both schemes - from $10^{10}$to 10^{13} s^{-1}

16. Conclusions

The key substantial fact underlying the above consideration is that an ensemble of free nuclei possesses remarkable properties as regards formation of a laser medium. These are the kinematic shift and the splitting of the spectral lines arising from radiative gamma-transitions, which result from the nuclear recoil in emission or absorption of hard photons. These properties open up favourable possibilities for staging the primary laser experiment. They

include the possibilities for the formation of hidden population inversion without excess of the number of excited nuclei over the number of unexcited nuclei, the realisation of broadband X-ray pump in unconventional unperturbing schemes, occurrence of anisotropic unidirectional stimulated emission without recourse to any reflecting structures that are hard to produce for the gamma-ray range, etc.

However, owing to the inhomogeneous Doppler line broadening inherent in an ensemble of free emitters, these promising possibilities open the way to the implementation of a laser medium with acceptable quantum amplification only when the «cooling» (monokinetisation) of the nuclear ensemble is so deep that the Doppler broadening is comparable with the intrinsic radiative width of a laser transition. Note that the two-quantum stimulated gamma-ray emission by the excited nuclei in oppositely directed photon beams is beyond the scope of the present consideration. In the process, obviating the adverse effect of Doppler broadening does not necessitate cooling of the nuclear ensemble at all [29, 30, 33, 77 - 80]. The nonlinear distributed feedback [81] arising in this case without the participation of reflecting or scattering structures results in an avalanche stimulated depopulation of excited states accompanied by a short high-intensity gamma-ray burst.

Eventually, the above-specified salient features of the radiative processes in the ensembles of free nuclei make it possible to conceptualise a primary experiment to observe nuclear quantum amplification of gamma-photons. The concept, at least with the present-day comprehension of the problem, is void of cardinal internal contradictions [24, 28, 29, 33, 78, 80, 82 - 85]. A sketch of the concept is provided by the diagram in section 14.

We specify three major problems along the path to this experiment:

* Selection of optimum candidate nuclides taking into account their nuclear and atomic properties on the basis of theoretical concepts and available data banks and, probably, pursuance of additional nuclear spectroscopic research.
* Development of the devices for the preparation, cooling, and confinement («traps») of the extended filamentary-shaped atomic beam of selected nuclides. Significant rise in atomic density in comparison with that attained to date.
* Development of the X-ray pump sources with equivalent blackbody temperatures of hundreds to thousands of kiloelectronvolts. Development of the devices for focusing and transporting the pump radiation to the nuclear beam.

It is pertinent to note that the last two problems refer to the branches of experimental physics that are now being developed intensively (independent of the nuclear gamma-ray lasing problem), which allows us to look forward to a relatively fast and successful solution of the problem. There remains the problem of developing efficient reflectors inthe subnanometer wavelength range. The application of such reflectors could radically reduce the requirements imposed on the nuclear density and the length of atomic beams.

These first-stage experimental problems do not refer in essence to nuclear physics and would not require a hot laboratory. Evidently, the results of these experiments would introduce significant corrections to the primary concept of a gamma-ray laser with free nuclei and the purposeful beneficial use of the kinematic spectral line shift arising from recoil. Therefore, only when their positive solution is obtained can a start be made on the planning and implementation of the primary gamma-ray lasing experiment with excited nuclei.

Table 5. Energy content of several isomers

Isomer	Transition energy keV	Lifetime yr	Energy content MJ/g	Decay period d
Nb^{93}	31	16.13	31	stable
Ag^{108}	109.44	418	100	$\sim 10^{-3}$
Ta^{180}	75	10^{15}	40	0.3
Bi^{210}	271.3	3×10^{6}	120	5

In an analysis conducted in 1988 of the feasibility of gamma-ray lasing assisted by the recoil of free nuclei, the author of Ref. [27] gave preference to the phononless solid-state approach, at least in the absence of a significant further improvement of the technology of cooling atoms. It seems as though, in light of the rapid advancement of the latter field in the past decade, along with progress in the development of new effective incoherent X-ray pump sources, we need to reconsider that judgement.

Needless to say, the proposed experiment, even if it is a success, would be of significance primarily for demonstration purposes. Hopefully, it would play the same role in the formation of *quantum nucleonics* as did the advent of a ruby laser in the history of laser physics in the visible range.

In conclusion we emphasise that an aspect of the nuclear gamma-ray lasers such as the problem of energy production also deserves consideration [43]. In essence, an exothermal chain nuclear reaction proceeds in a gamma-ray laser dependent on the employment of long-lived metastable isomers. The understanding of this fact was evident in the very first proposals of a gamma-ray laser dating as far back as 1961 [6], along with the Technical Report (supposedly the first in this field) No. 74-2104 by the Research Institute of the State Committee of the Council of Ministers of the USSR on Radioelectronics entitled «On the Feasibility of a Chain Reaction of Stimulated Radiative Transitions of Excited Nuclei» (1961) (see Ref. No. 5 in Ref. [86]). Indeed, the specific energy stored in a series of isomers is ~ 100 MJ/g (see the examples in Table 5). This is approximately two orders of magnitude lower than the energy content of fissionable materials but exceeds the efficiency of a hydrocarbon fuel by three orders of magnitude. The advantages and disadvantages of this intermediate figure determine the rank of a laser nuclear chain reaction in the hierarchy of energy production. The main argument in its favour is, of course, ecological in character and resides in the absence of long-decaying radioactive wastes.

Finally, in contrast to the purely practical problem of energy production, mention can be made of an exotic problem that falls outside the scope of laboratory experiments. We are dealing with the possible role of the natural stimulated emission of gamma-ray quanta involving excited free nuclei and antiparticles in cosmological and astrophysical phenomena («cosmic gamma-ray lasers») [2, 87, 88]. The latter remark closes the circle: A.Eddington's statement opening this paper was borrowed from precisely the astrophysical monograph [2], which was supposedly the first step in the application of Einstein's laws of stimulated emission to nuclear processes.

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